

Produktintegration zweier trigonometrischer Funktionen

Bestimmen Sie $\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx$

$$u(x) = \sin(x) \quad u'(x) = \cos(x)$$

$$v(x) = \sin(x) \quad v'(x) = \cos(x)$$

$$\text{Zur Erinnerung } \int u' \cdot v = [u \cdot v] - \int u \cdot v'$$

Wir addieren

$$\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [\sin(x) \cdot \sin(x)]_0^{\frac{1}{2}\pi} - \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx \quad | \quad + \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx$$

$$2 \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [\sin(x) \cdot \sin(x)]_0^{\frac{1}{2}\pi}$$

$$2 \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [\sin\left(\frac{1}{2}\pi\right) \cdot \sin\left(\frac{1}{2}\pi\right)]_0^{\frac{1}{2}\pi} = [\sin\left(\frac{1}{2}\pi\right) \cdot \sin\left(\frac{1}{2}\pi\right) - \sin(0) \cdot \sin(0)]$$

$$2 \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [\sin\left(\frac{1}{2}\pi\right) \cdot \sin\left(\frac{1}{2}\pi\right)]_0^{\frac{1}{2}\pi} = 1 \cdot 1 - 0 \cdot 0 \quad |:2$$

$$\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [\sin\left(\frac{1}{2}\pi\right) \cdot \sin\left(\frac{1}{2}\pi\right)]_0^{\frac{1}{2}\pi} = 1 : 2$$

$$= \frac{1}{2}$$

Bestimmen Sie $\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx$

Nun aber mit folgender Einteilung:

$$u(x) = -\cos(x) \quad u'(x) = \sin(x)$$

$$v(x) = \cos(x) \quad v'(x) = -\sin(x)$$

$$\text{Zur Erinnerung } \int u' \cdot v = [u \cdot v] - \int u \cdot v'$$

$$\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [-\cos(x) \cdot \cos(x)] - \int_0^{\frac{1}{2}\pi} -\cos(x) \cdot (-\sin(x) dx)$$

$$\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [-(\cos(x))^2] - \int_0^{\frac{1}{2}\pi} \cos(x) \cdot \sin(x) dx \quad | \quad + \int_0^{\frac{1}{2}\pi} \cos(x) \cdot \sin(x) dx$$

$$2 \cdot \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = [-(\cos(x))^2]$$

$$2 \cdot \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = -(\cos\left(\frac{1}{2}\pi\right))^2 - (-\cos(0))^2$$

$$2 \cdot \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = -(0)^2 + 1^2$$

$$2 \cdot \int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = 1 \quad |:2$$

$$\int_0^{\frac{1}{2}\pi} \sin(x) \cdot \cos(x) dx = \frac{1}{2}$$